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PARACONICAL PENDULUM AS A DETECTOR OF GRAVITATIONAL EFFECTS DURING SOLAR ECLIPSES (PROCESSING DATA AND RESULTS)

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A statistical analysis of data collected during observation of a total solar eclipse of July 11, 1991 at Mexico City is presented. The variation of the velocity of the azimuth of the major axis of the pendulum's oscillation ellipse during the eclipse is determined and the sudden deviation of the azimuth of the plane of swinging at the start of the eclipse is calculated.

It has already been noted [1] that Allais had conducted a continuous series of observations for several weeks, and had then constructed a graph representing the dependence of the azimuth of the plane in which the pendulum swings on time [2]. By means of harmonic analysis using the filter, period pattern, and correlation pattern method), he identified the periodicity of the variation of the azimuth (with periods of 12, 24, and 25 h) with a total effect on the order of the Foucault effect. During the solar eclipse of June 30, 1954 at Paris he observed the variation of the azimuth of the plane in which the pendulum swings during the eclipse, noting a sharp deviation of the azimuth by 5° at the time of first contact (start of eclipse) and maximal deviation of 15° for 20 min before the maximum of the full phase of the eclipse. During the eclipse of October 2, 1959 Allais found the deviation of the azimuth of the plane at the full phase maximum to be only 6°, though the area of the solar surface covered was only half as great.

All our experiments differed from those of Allais in terms of two essential features. First, because the pendulum had been placed in a constant-temperature vacuum and the fact that the pendulum was under remote control, it was not possible to observe the azimuth with the naked eye or to take readings of the azimuth. Second, because the pendulum was launched, halted, and oriented automatically in the same start plane of swinging, we had been unable to "glue" together the values of the azimuth into a continuous graph of its variation over time until the advent of computerized registration systems.

Therefore, a method of photographic recording was used in 1961. Photographs depicting the motion of a pendulum along the same path every 2 h for three days were compared. Because of inaccuracies in the design of the instrument, no observations during the eclipse were obtained, though we were able to identify the Foucault effect the day before the eclipse on the basis of the shape of a curve drawn in the photographic film by the motion of a light spot.

In the pendulum instrument which we created for making observations during the eclipse of July 22, 1990 at Belomorsk, a method of opto-electronic registration was employed solely for monitoring the launch time of the pendulum. Nevertheless, the graphs of the two-day cycle of experiments which we obtained served as valuable data for further improvement of the data registration and collection system. We again made use of photographs in order to qualitatively compare the motion of the pendulum. We identified three automatic remote launches of the pendulum of identical two-hour duration with the same initial azimuth, the laundry occurring within an insulated chamber with internal pressure 20 mm Hg. The first launch occurred 10 h prior to the eclipse, the second 10 min before it began, and the third 2 h after it had concluded.

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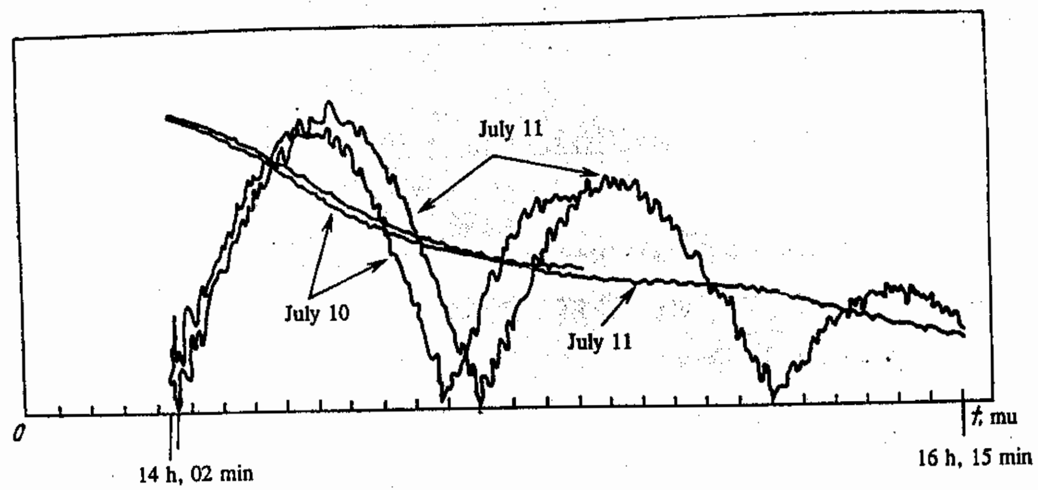

Fig. 1. Graphs of the length of the major and minor semiaxes of the ellipse as a of swinging function of time, all for the third series of experiments completed the day before the eclipse and the day of the eclipse, with the graphs combined by means of superposition.

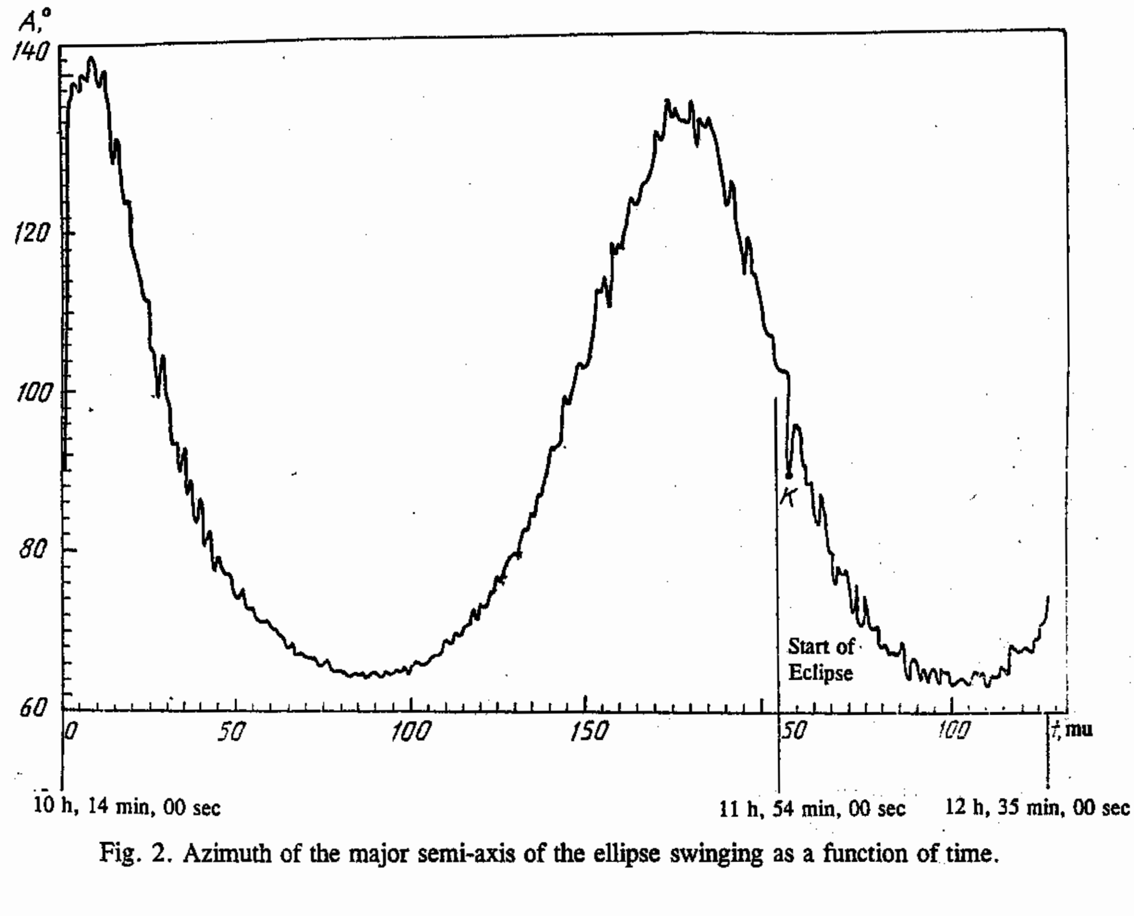

Fig. 2. Azimuth of the major semi-axis of the ellipse swinging as a function of time.

The three photographs obtained may be compared only qualitatively, since because of the absence of any timer device the light spot created an area of density (continuous record) in the photographs the height of which characterized the amplitude of the pendulum swinging, while the length gave the rate of displacement of the pendulum's plane of swinging. The more slowly the pendulum traveled, the greater the density of the spot in the photographs. Reference photographs taken before and after the eclipse proved to be entirely identical, whereas the photograph taken during the eclipse proved to be quite different. It does not seem possible to make more detailed conclusions though such attempts have been under taken.

TABLE 1

Date	Series	Length of Period, min			
		T_1	T_2	T_3	T_1+T_2
5.05.91	1	37	—	—	—
	2	—	40	33	78
6.05.91	1	37	39	32	76
	2	42	39	28	81
	4	37	43	33	80
7.05.91	1	38	44	32	82
	2	39	42	33	81
	3	39	39	—	78
Mean Period		38±2	41±2	32±2	79±2

TABLE 2

Date	Experiment	Series	Length of Period, min			
			T_1	T_2	T_3	T_1+T_2
10.07.91	1	1	—	—	38	84
	2	2	42	—	...	—
	3	3	43	43	40	86
	4	4	43	43	38	86
11.07.91	5	1	38	43	—	81
Mean Period			41±3	43±1	39±2	84±3
11.07.91	5	1	—	—	46	—
	6	2	45	—	—	—
	7	3	44	51	—	95
	8	4	41	47	43	88

A total of 19 series of laboratory experiments were completed in anticipation of the experiment that was planned for the solar eclipse of July 11, 1991 at Mexico City. All the conditions of the field experiment (automation, placement of the pendulum in a constant-temperature chamber, opto-electronic registration, and computerized collection and recording of data), other than placement of the pendulum in a vacuum chamber, were simulated, although the pendulum itself was placed in an air-tight housing. In the course of two days of observations made during the expedition a total of eight series of experiments (four on the day before the eclipse and four on the day of the eclipse were performed). Prior to the experiments the pressure in the housing was reduced down to 32 mm Hg by means of a vacuum pump.

Information written on 5-inch diskettes was then processed in Italy and at Moscow by two independent methods. The Italian workers calculated the parameters characterizing the motion of the pendulum by means of Fourier analysis. The values of 30 parameters, such as the area of the ellipse of swinging, length of the major and minor semi-axes, deviation of the center of the ellipse from the origin as measured along the x- and y-axis, and so on were represented graphically as functions of time. The graphs of corresponding series for July 10 and 11 (day of the eclipse) were compared.

Graphs of the length of the major semi-axis of the ellipse of swinging (descending curve) and of the minor semi-axis (analog of parabola from zero to zero) as functions of time for the third series of July 11 are shown combined together in Fig. 1 by means of the method of superposition. This series of experiments was completed following the full phase of the eclipse until the eclipse had ended and also for the corresponding third series of July 10, the day before the eclipse. Time is laid out along the horizonal axis (with markers every 5 min), and the length of the semi-axes in conditional units along the vertical axis. It is clear that the length of the major semi-axis decreases with time due to decay of the swinging, while the length of the minor semi-axis grows from zero (the initial straight line) to a maximum (widest ellipse) and then decreases again down to zero (degeneracy of ellipse into a straight line) at the moment the motion of the plane in which the pendulum swings reverses direction.

The Moscow group focused their efforts on calculating the azimuth of the major semi-axis of the ellipse of swinging by the method of least squares with respect to the x- and y-coordinates of points of the trajectory of the approximating ellipse.

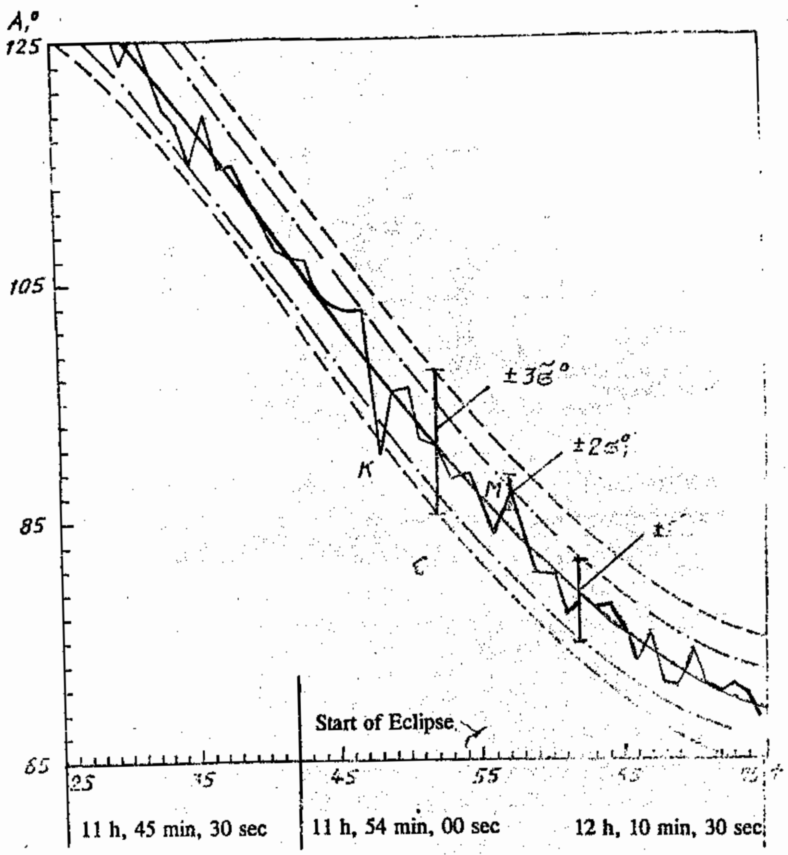

Fig. 3. Portion of graph in Fig. 2 with the scale magnified and with error bands $\pm 2\sigma$ and $\pm 3\sigma$.

The displacement of the pendulum's ellipse of swinging with periodic reversal of the direction of this displacement (see Fig. 3 of [1]) is clearly evident in all the resulting graphs of the variation the azimuth as a function of time. Depending on the length of the series (from 1 to 4 h), one to seven periods between the extreme positions of the plane of swinging were obtained. All the graphs of the laboratory series were identical to the graph shown in Fig. 2 given in [1], which demonstrates the stability of the instrument. A typical graph was shown here in Fig. 2 as an example demonstrating the processing of the experimental observations (first series of July 11). The scale factor on the horizontal axis is equal to 5 min, and that on the vertical axis, 2°.

Investigation of Periodicity. From the nature of the graphs it is clear that it makes sense to work with the first three periods, the most reliable of which is the second period, when the pendulum's motion is the most stable. The initial launch conditions always influences the length of the first period, though this occurs automatically. The third period, on the other hand, is less reliable due to difficulties involved in determining the ellipse's semi-axes, inasmuch as the spread of the pendulum's actual swings reaches only 4 mm by that time. However, the sum of the first two periods $T_1 + T_2$ from the first maximum to the second may prove to be a rather good characteristic (cf. Fig. 2 of [1] and Fig. 2 in the present article) even if T_1 is rather difficult to discriminate from T_2 due to malfunctioning in the electronic instruments or other causes, as a result of which noise is sometimes encountered in the recording instruments.

Using only absolutely reliable values of the periods T_1, T_2, T_3 and $T_1 + T_2$ from the corresponding series of experiments, the results may be presented in the form of a table and the mean values of these periods calculated. Table 1 presents the results of laboratory experiments during which information arrived at the computer every minute. The experiments were conducted with the instrument's housing kept closed but not pumped down to a vacuum. Table 2 shows the results of calculations of the periods on the basis of observations made during the expedition to Mexico the day before the eclipse of July 10, 1991 and on the day of the eclipse. The experiment was conducted using a pendulum instrument placed in a fully air-tight chute, maintained at a constant temperature, and evacuated down to a vacuum which was itself in a special underground laboratory at a depth of 20 m. Information arrived at the computer every 30 sec. During the first experiment it was practically

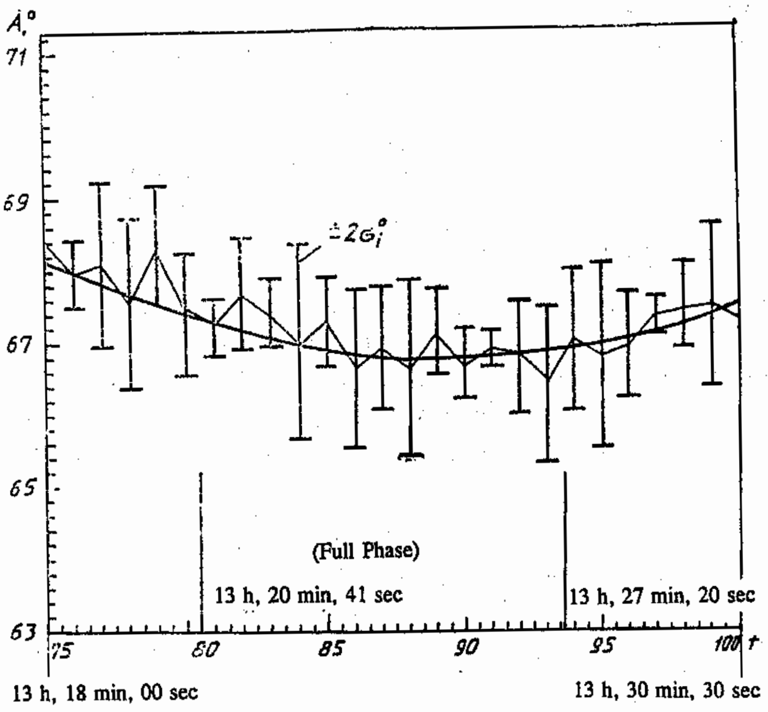

Fig. 4. Azimuth of the major semi-axis as a function of time at the moment of full phase of the eclipse.

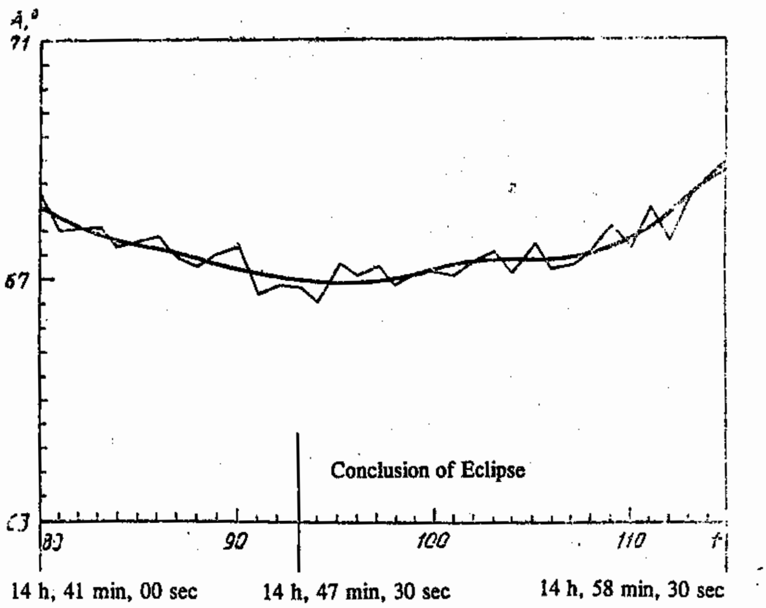

Fig. 5. Azimuth of major semi-axis as a function of time at the moment the eclipse concluded.

impossible to distinguish the transition from period T_1 to period T_2 due to malfunctioning of the electronic recording instrument. During the second and sixth experiments only T_1, now of shortened duration, was measured, because the total phase of the eclipse occurred in the sixth experiments. It was impossible to measure T_3 in the seventh experiment, inasmuch as the pendulum had halted and then restarted in connection with the end of the eclipse.

Investigation of Singular Points. The shape of the graphs demonstrates not only the cyclic nature of the variation in the azimuth of the plane of the pendulum is swinging, but also the existence of constant and more or less significant "noise."

A total of 178 graphs in 21 series of laboratory and expedition experiments were renewed. Sufficiently significant deviations from the main trend against the total noise background was investigated. It is necessary to determine the extent to which these deviations are random and whether some of them are connected to the solar eclipse itself.

Singular Points at the Moments of Contact. Recall that the moments of contact refer to the start of the apparent occultation of the surface of the Sun by the Moon (first contact), the start of the total phase of an eclipse (second contact), the end of the total phase (third contact), and the end of the apparent occultation of the surface of the Sun (fourth contact).

The greatest deviation (12°) occurred precisely 2.5 min after the start of the eclipse (first contact), or point K on the graph. The graph of this first series of July 11 is presented in Fig. 2. Each discrete measurement of the azimuth (point on the graph) was carried out using the method of least squares with a corresponding error. The maximal error σ_i was 4.27°. The Arafer program used for processing does not allow us to automatically construct the standard deviation, therefore an approximation by means of a polynomial of degree 10 was applied. Figure 3 depicts part of the graph in Fig. 2 with a sample of 45 points (n = 45). If we let $y_i°$ denote the y-coordinate of each point of the graph with the value of $\sigma_i°$ corresponding to it, and let $y_i°$ be the corresponding value of the y-coordinate on the approximating curve, we will have

$$\tilde{\sigma}^{0} = \frac{1}{n}\sum_{i=1}^{n} \sigma_i^0, \quad \tilde{\sigma}^{a} = \left[\frac{\sum_{i=1}^{n} (y_i^0 - y_i^a)^2}{n}\right]^{1/2}, \tag{1}$$

where $\tilde{\sigma}^a$ is the standard deviation for the approximating curve. If $\tilde{\sigma}° \geq \tilde{\sigma}^a$, the method of approximation is well defined. For the sample of the graph in Fig. 3, n = 45, $\tilde{\sigma}° = -2.1045°$, and $\tilde{\sigma}^a = 1.7228°$, i.e., the method of approximation is well defined. The graph shows the approximating curve, error band $\pm 2\tilde{\sigma}°$, and error band $\pm 3\tilde{\sigma}°$. The singular point K we are investigating is beyond the band $\pm 3\tilde{\sigma}°$.

It was hypothesized that this sharp deviation of the azimuth is no accident. The criterion for verifying the hypothesis was chosen so that the probability P of refuting the hypothesis was low when the hypothesis was true. It was necessary to calculate the significance level of the probability P for which a standardized variable U could be used, inasmuch as in this case n > 30 and the distribution may be considered normal ($U = \frac{X - \mu}{\sigma}$ with null mean $\mu = 0$ and unit standard deviation $\sigma = 1$). Setting $\Delta y = y_i^o - y_i^a$

$$\Delta\tilde{y} = \frac{1}{n}\sum_{i=1}^{n} \Delta y_i, \quad s = \left[\frac{\sum_{i=1}^{n} (\Delta y_i - \Delta\tilde{y})^2}{n-1}\right]^{1/2}, \quad U = \frac{\Delta y_i}{s}, \tag{2}$$

we find that for n = 45, $\Delta\bar{y} = 0.06$ and s = 1.74. At the point K$|\Delta y_i| = 6.4$, whence U = 3.7. From special tables which tabulate the values of P as a function of U, the corresponding value of P was found to be 0.00022. Thus, the probability that the deviation of the azimuth at the moment the eclipse commences proved to be random, equal to $2 \cdot 10^{-4}$ or 0.02%.

For the sake of comparison, let us verify point K of the graph of Fig. 3. For this point we find that $\sigma_i° = 0.7272$, $|\Delta y_i| = 2.99$, U = 1.67, and, correspondingly, P = 0.09. The probability that the deviation at the point M is random is $9 \cdot 10^{-2}$, or 9%, i.e., 1000 times greater than at the point K. Thus, the main criterion for the existence of singular points is not that the approximating curve falls inside the error band $\pm 2\sigma_i°$, but that the point exists outside the band $\pm 2\tilde{\sigma}°$ relative to the line of approximation (see Fig. 3).

The graph showing how the azimuth corresponding to the second series of July 11 varies (it is during this series that the full phase of the eclipse occurs) does not demonstrate any major features. Nevertheless, we renewed a sample with n = 54 (this part of the graph is shown in magnified scale in Fig. 4, with maximal $\sigma_i°$ equal to 0.67°). All the deviations in this graph, including those points at which the approximating curve does not extend into the error band $\pm 2\sigma_i°$, fit completely inside the band $\pm 2\tilde{\sigma}°$ for the approximating curve.

There were no apparent major changes on the graph representing the results from observations in the third series of July 11 (it was during this series that the eclipse concluded). From a detailed examination of the part of the graph around a point corresponding to the conclusion of the eclipse (fourth contact) depicted in magnified scale, it is clear that the approximating curve which we constructed from a polynomial of degree 10 is shifted above the line of the graph prior to the moment of contact. Following the moment of contact the approximating curve falls below the line of the graph and outside the error bands. Therefore, a sample of values y_i of the azimuth for n = 50 with center at the point of contact was computed. The maximal value $\sigma_i°$ within the sample was 0.75°. The mean error $\tilde{\sigma}° = 0.33$, and all the points of the sample were inside the band $\pm 3\tilde{\sigma}°$ constructed on the approximating curve, though points in the graph before and after the moment of contact did not fall in the band $\pm 2\tilde{\sigma}°$. Under the assumption that the peripheral zones of the graph exerted a strong influence in the approximation, a new curve was constructed taking into account only nearby zones. The result is shown in Fig. 5, in which all the points of the graph are in the band $\pm 2\tilde{\sigma}°$.

Singular Points Other Than at Moments of Contact. Points of this type included points of the graph for those series of the experiment of July 10 and 11 conducted the day before the eclipse prior to and following the eclipse and in which quite marked deviations of the azimuth of the plane of swinging (on the other of 3-6°) were obtained. For those points in the sample at which the maximal deviation of the azimuth was observed (third series of July 11), a sample n = 34 with center at this point was constructed. The maximal value of $\sigma_i°$ was 3.12°. It was found that $\tilde{\sigma}° = 1.5$ in the course of the calculations and, thus, all points in the sample, other than the center point, were inside the band $\pm 2\tilde{\sigma}°$. The center point was a bit outside the band $\pm 3\tilde{\sigma}°$, hence it was necessary to apply the same hypothesis as in the preceding case. As a result of calculations, it was found that $\Delta\bar{y} = 0.66$ and s = 1.13, and, since for this point $\Delta y_i = 2.4$, the standardized random variable U = 2.1. From the tables it was found that, correspondingly, P = 0.04. Consequently, the probability that the deviation of the azimuth at this point occurred randomly is 4%.

Other points of this type for the eight experimental series were also verified. In all the samples the deviations of the azimuth fell within the error band $\pm 2\tilde{\sigma}°$ of the approximating curve.

Malfunctioning Points. Deviations at times corresponding to the points of the graph used earlier were characterized by the fact that the natural error of the computed points to which they corresponded was sufficiently small by comparison with other computed points in its neighborhood (i.e., in the area around these points), and that the approximating curve was not in the band of this error.

However, there are points on the graphs that would appear to indicate malfunctioning of the electronic recording instrumentation. The most typical of such cases is the point on the graph corresponding to the first series of July 10. For this reason, we considered a sample n = 34 with center at this point. The natural error of the computed azimuth at this point, $\sigma_i° = 5.83°$, is anomalously high by comparison with adjacent points ($\sigma_{i-1}° = 0.23$; $\sigma_{i+1}° = 1.07$). In the computations we found that $\tilde{\sigma}° = 0.56$, $\tilde{\Delta}y = 0.03$, $\tilde{\sigma}^a = 0.82$, and since $\tilde{\sigma}° < \tilde{\sigma}^a$, the approximation is not well defined.

Results. By comparing the graphs which represent the variation of the minor semi-axis of the swinging ellipse (see Fig. 1), we may conclude that during an eclipse the ellipse creation cycle, starting from degeneracy in a straight line (minor semi-axis equal to zero) through a maximum (maximum length of minor semi-axis), to the next degeneracy (minor semi-axis again zero), exhibits a 5-min delay. The same effect is observed in all the other graphs of the characteristic parameters calculated by our Italian colleagues by means of Fourier analysis.

By studying Tables 1 and 2 we are led to note the following: an increase in the length of the periods in the field experiment in Mexico City by comparison with the laboratory series attributable to the fact that the instrument had been placed in a vacuum; a marked increase in all three periods of the variation of the azimuth of the plane in which the pendulum swings during the eclipse (T_1 by 3.5 min, T_2 by 7 min, and T_3 by 7 min, on average); a trend towards a reduction in all three periods after the eclipse (the value of T_1 was comparable with its value before the eclipse, white T_2 and T_3 were less by 4 and 3 min, respectively, but were still not comparable with their values before the eclipse).

From the graphs and the analysis it is clear that the sharp deviation of the azimuth of the plane in which the pendulum swings by 12° at the start of the eclipse (first contact) is noteworthy.

Discussion of Results. The experiments we conducted to observe the motion of paraconical pendulums during solar eclipses in 1961, 1990, and 1991 differed from Allais' experiments of 1954-1955 and 1958-1959 not only in terms of the construction of the pendulums, but also in terms of methodology. We did not consider the piecewise-continuous time dependence of the azimuth of the plane in which the pendulum swings over the course of several months, as did Allais, but instead investigated the behavior of the pendulums under laboratory conditions with pendulums that were all launched at the same initial azimuth (from the standpoint of stability) along the same path. Next, all the experiments were repeated under field conditions and the control series were compared with the series carried out during the eclipses. Therefore, we were still unable to verify Allais' claim that there is a diurnal periodicity in the motion of a paraconical pendulum.

In contract, it would appear that the pulsed deviation of the azimuth of the plane in which the pendulum swings at the moment the eclipse starts has been confirmed. Indeed, while the conditions of Allais' experiment were, to put it mildly, far from ideal, the stability of the instrument used in the observations in Mexico had been proved by numerous laboratory tests.

Moreover, because the doings of the experiment was sufficiently well-defined (including an underground laboratory at a depth of 20 m, isolated rooms use of remote control, the creation of a vacuum, thermostatically controlled conditions, and computer-based registration), there is reason to hope that future investigations will be successful.

The existence of fluctuations in the rate of variation of the azimuth of the plane in which the pendulum swings during an eclipse, obtained independently in Italy and Moscow by two different methods of data processing (Fourier analysis and the method of least squares), is a far more interesting result. This rate is easily computed by dividing ΔA_i (variation of azimuth from one degenerate ellipse to the next, measured in degrees) by T_1 (time it takes for the plane of swinging to pass from one degenerate ellipse to the next, i = 1, 2, 3). Before the start of an eclipse its mean value $\bar{v}_1$ corresponding to the period $\bar{T}_1$ of motion of the plane of swinging in the same direction as the Foucault effect amounted to 1.9°/min. The mean rate $\bar{v}_2$ (for $\bar{T}_2$) as the plane of swinging moves in the direction the Foucault effect was 1.7°/min. The mean rate $\bar{v}_3$ (for $\bar{T}_3$) as the plane of swinging again moves in the same direction as the Foucault effect was again 1.9°/min. At the latitude of Mexico City the Foucault effect was roughly 0.1°/min. Consequently, the constant component V of the velocity $\bar{v}_i$ (i = 1, 2, 3), attributable to the construction of the instrument and other factors was 1.8°/min, since $\bar{v}_1 = V + V_F = 1.8 + 0.1 = 1.0$; $\bar{v}_2 = V - V_F = 1.8 - 0.1 = 1.7$; and $\bar{v}_3 = V + V_F = 1.8 + 0.1 = 1.9$. During the eclipse the rates v_1, v_2, and v_3 were 1.9°/min, 1.2°/min, and 1.6°/min, respectively. Thus (see Table 2), right after the start of the eclipse the constant component V dropped by 0.3°/min and then, continuing to drop, fell to 1.2°/min. The maximal drop was 0.5°/min, i.e., it was five times as great as the Foucault effect. Four hours after the end of the eclipse v_1 = 2.0°/min, v_2 = 1.4°/min, and v_3 = 1.6°/min, respectively, i.e., the pendulum nevertheless did not return to the usual stable position.

Findings. 1. A new type of instrument distinguished by a high degree of stability and making it possible to register effects on the order of the Foucault effect (10-6 g) was created.

2. A novel technique for making dynamic observations during solar eclipses using a paraconical pendulum as sensing element was developed.

3. Results that do not contradict other analogous experiments were obtained.

4. A series of repeated experiments with several identical instruments situated along the line of full eclipse at distances on the order of several hundred kilometers is needed.

5. The results of observations in Mexico City are still difficult to interpret in unambiguous fashion [8], i.e., it would be useful to continue theoretical studies and laboratory experiments.

Conclusions. Experiments using other instruments were also conducted during observations of the solar eclipse of July 11, 1991 in Mexico. Denis [10] discovered a variation of the rate of rotation of the Foucault pendulum's plane of swinging. A group of Belgian researchers discovered a 1.5 mG variation in the gravitational force working with LaCoste–Romberg tidal gravimeters [10]. In discussing these and other results participants of the 74th session of the International Geodynamic Seminar at Luxemberg decided to undertake a coordinated international expedition to observe the solar eclipse that was to occur on November 3, 1994 in South America. The results have been processed and will be published in a later issue of this journal.

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